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Bayesian factor analysis

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CHAPTER 1 INTRODUCTION

As is well known, factor analysis is a mathematical model which attempts to explain correlations between a large set of p variables in terms of k underlying factors ($k < p$). These factors are considered as latent variables. If factor analysis is not merely regarded as a data analytical method but as a technique which is useful to judge the adequacy of the presupposed model in the population, a formal statistical theory is required. Statistical inference in factor analysis is discussed in detail in Anderson & Rubin (1956). It is said that the k -factor model holds in the population if every variable in a p -dimensional random vector can be taken as an addition of 1) a linear combination of k common random factors, 2) a unique random factor and 3) a constant. The common factor weights are collected in a $p \times k$ matrix which is called the factor-pattern matrix.

With additional assumptions (see section 2.1), the validity of the k -factor model can be expressed in a decomposition of the $p \times p$ population variance-covariance matrix in terms of three matrices. These matrices are the factor-pattern matrix Λ , the $k \times k$ correlation matrix of the common factors Φ and the $p \times p$ diagonal variance-covariance matrix of the unique factors Ψ . The decomposition is of central importance in the present thesis.

The factor model, however, exhibits indeterminacies.

1. If the k -factor model holds then the $(k+1)$ -factor model holds (see Reiersøl, 1950). This problem is usually solved by trying to determine the minimal value for k for which the decomposition exists.
2. If the k -factor model holds given k , different solutions for Ψ may exist. However, if k is less than a well-known upper bound (see section 2.1), identifiability of Ψ is generally to be expected (see Reiersøl, 1950).
3. If the k -factor model holds given k and Ψ , different solutions for the pair (Λ, Φ) exist (rotational indeterminacy).

4. If the k-factor model holds given k , Ψ and (Λ, Φ) , different solutions exist for the common and unique factors (factor score indeterminacy).

It should be noted that in the decomposition the fourth indeterminacy is not relevant. Determination of factor scores is thus not considered in the present study. Usually, the interpretation of the factor-pattern matrix is the essential part of a factor analysis.

The point of view in the present study is that the indeterminacies are due to a lack of a sound theoretical framework in which a factor analysis should be embedded. Moreover, factor analysis acquires its relevance in the interpretational stage in which an escorting theory is required. The opinion of Elffers, Bethlehem and Gill (1978) is adopted here: "Finally the interpretational stage is meant to make plausible that a real meaning, as seen from the science under discussion, can be attached to the mathematically possible factor. So an underlying theory is wanted, which may be vague or rudimentary, but should be powerful enough to discuss the tenability of proposed interpretations. For only by defending or rejecting a certain interpretation factor analysis can become a valuable research tool whereas on the contrary an analysis without an interpretational stage ought to be considered as hardly more than an exercise in computation" (p. 183). Elffers et al. further note that the rotational indeterminacy can be overcome only if there is an underlying theory strong enough to be able to maintain or reject a given solution (pp. 188 - 189). According to the recommendations of Armstrong (1967) a particular investigator should make prior evaluations before conducting a factor analysis. These evaluations should be in terms of the applicability of the factor model, the expected number of factors, the types of factors, the consideration of the variables and the expected relationships among the factors.

Obviously, the validity of the factor model is tested on sample

results, such as in maximum likelihood factor analysis (MLFA; see Lawley & Maxwell, 1971; Jöreskog, 1967, 1969). MLFA is based on the assumption of multinormality of the variables. In a Bayesian approach to factor analysis, a prior joint density can be specified for the factor parameters that occur in the matrices in which the variance-covariance matrix is decomposed. The likelihood function is then multiplied by this prior density to obtain a posterior joint density of the factor parameters. Press (1972, p. 317) points out that the use of informative prior densities can eliminate the rotational indeterminacy. Martin and McDonald (1975) discuss the applicability of a prior informative density for Ψ to prevent the frequently observed Heywood cases (negative diagonal elements of Ψ). In the present Bayesian approach, informative densities for the factor parameters are specified. These densities are based on substantive theoretical considerations regarding a particular research project. Prior theoretical knowledge of the parameters can be incorporated in the prior densities. Partially as a consequence of the nature of the factor parameters a reliable assessment of this knowledge is a difficult matter. Although Λ is generally used for interpretative purposes, the interpretation of its elements is difficult. It will however be seen in section 2.2 that the $p \times p$ variance-covariance matrix can be decomposed in a reparameterized form. It is then composed in 1) the $p \times p$ diagonal matrix with standard deviations of the variables 2) the $p \times k$ matrix with correlations between the factors and the variables and 3) the $k \times k$ factor correlation matrix Φ . The second mentioned matrix, which is very useful for interpretative purposes, is called the correlation-structure. Compared with pattern coefficients, prior information regarding correlation coefficients is easier to assess. Thinking in correlational parameters is common to a factor analyst who uses explorative factor analysis techniques on the sample correlation matrix of the variables. Therefore, it is assumed here that existing prior information regarding the parameters in the three matrices can be assessed and incorporated in prior informative

densities. The present choice of the prior densities is chiefly based on simplicity, convenience and familiarity. Furthermore, the identification problem can be resolved by specifying prior densities with identified modes (see Drèze, 1977). Independent normal densities and independent inverse chi densities are assumed for the correlation coefficients and for the p standard deviations, respectively. This particular choice is further discussed and worked out in the next chapter. The assignment of the values to the distributional parameters is highly facilitated by an interactive computer program, written by the authors. In this program, called BAYFAC-I (Bayesian Factor Analysis-Interactive), sophistication of a particular user regarding the factor parameters is enhanced by means of interrogative questioning. Another essential part of the program is the adaptation of initial specifications which are improper (in the sense that more than slight nonzero probabilities are assigned to inadmissible parameter vectors).

Combining the inextricable likelihood function with the quite simple joint prior density, a complex posterior density is the result, in which it is too difficult to obtain posterior marginal densities or expectations. However, posterior modal estimates can be obtained by using an optimization procedure. The assigned values to the prior modes are thus directly comparable to the obtained posterior modal values. The parallel with the maximum likelihood approach in using maximizing values as the Bayesian estimates is self-evident. The posterior density is optimized here by means of a non-interactive program, called BAYFAC-O (Bayesian Factor Analysis-Optimization), also written by the authors.

This chapter concludes with an overview of the coming chapters. Chapter 2 presents the complete reparameterized Bayesian factor model. Some of the remarks in this introductory chapter are considered in detail and other Bayesian approaches to factor analysis are discussed. Useful mathematical deductions for the

optimization are given in the appendix of the chapter. Chapter 3 gives a detailed description of the acquiring, checking and adjusting of the prior information. Illustrations of BAYFAC-I are given in the appendix.

Chapter 4 presents two examples of applications of BAYFAC-I and BAYFAC-O. The uniqueness of the solutions is illustrated and the relative weights of the likelihood and the prior density are discussed in an example.